The QCD Analysis of the Structure Functions and Effective Nucleon Mass.

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Abstract

On the basis of the target mass corrections to structure functions of deep-inelastic scattering of leptons, we evaluate effective nucleon mass that turns out to be twice $M_{nucl.}$ for deep-inelastic scattering on the nucleus target and equals $M_{nucl.}$ for the hydrogen target.

Deep-inelastic scattering of leptons provides a precise information on structure functions (SF) of a nucleon. It is well known that when target mass corrections (TMC) are taken into account, the QCD description of the SF of deep-inelastic scattering is improved. These effect is of the order $M^2_{nucl.}/Q^2$. In this aricle, we are going to consider the question wherher the mass of a nucleon is the best value for the description of data or in order to make the fit better, one has to use another value $M^{eff.}$ which could differ from the mass of nucleon.

The Nachtmann moments [1] of SF F_2 and F_3 are defined as:

$$M_2^{QCD}(N,Q^2) = \int_0^1 \frac{dx\xi^{N+1}}{x^3} F_2(x,Q^2) \frac{3+3(N+1)V+N(N+2)V^2}{(N+2)(N+3)}, \tag{1}$$

$$M_3^{QCD}(N,Q^2) = \int_0^1 \frac{dx\xi^{N+1}}{x^3} F_3(x,Q^2) \frac{3+(N+1)V}{(N+2)}, \tag{2}$$

where

$$\xi = 2x/(1+V), \qquad V = \sqrt{1+4M^2_{nucl.}x^2/Q^2} \tag{3}$$

Equations (1,2) could be expanded into a series in powers of $M^2_{nucl.}/Q^2$. Retaining only the terms of the order $M^2_{nucl.}/Q^2$ one could obtain:

$$M_2(N,Q^2) = M_2^{QCD}(N,Q^2) + \frac{N(N-1)}{N+2}\frac{M^2_{nucl.}}{Q^2} M_2^{QCD}(N+2,Q^2), \tag{4}$$

$$M_3(N,Q^2) = M_3^{QCD}(N,Q^2) + \frac{N(N+1)}{N+2}\frac{M^2_{nucl.}}{Q^2} M_3^{QCD}(N+2,Q^2). \tag{5}$$

$M_2(N,Q^2)$ and $M_2(N,Q^2)$ are the Mellin moments of the measured SF F_2 and xF_3:

$$M_2(N,Q^2) = \int_0^1 dx x^{N-2} F_2(x,Q^2), \tag{6}$$

$$M_3(N,Q^2) = \int_0^1 dx x^{N-2} xF_3(x,Q^2), \tag{7}$$

$$N = 2,3,...$$

The Q^2 - evolution of the moments $M_2^{QCD}(N,Q^2)$ and $M_3^{QCD}(N,Q^2)$ is given by QCD [2, 3]. For the nonsinglet SF:

$$M_3^{QCD}(N,Q^2) = \left[\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\right]^{d_N} M_3^{QCD}(N,Q_0^2), \tag{8}$$

$$N = 2,3,...$$

$$d_N = \gamma^{(0),N}/2\beta_0, \qquad \beta_0 = (11-\frac{2}{3}f).$$

$$\alpha_s(Q^2)/4\pi = 1/\beta_0 \ln(Q^2/\Lambda^2_{LO}) \tag{9}$$

$$\gamma_N^{(0)NS} = \frac{8}{3}[1 - \frac{2}{N(N+1)} + 4\sum_{j=2}^{N}\frac{1}{j}] \ . \tag{10}$$

The unknown coefficients $M_3(N, Q_0^2)$ in (9) could be parametrised as a Mellin moments of some function:

$$M_3^{QCD}(N, Q_0^2) = \int_0^1 dx x^{N-2} A x^b (1-x)^c (1+\gamma x), \quad N = 2, 3, ... \tag{11}$$

where constants A, b, b and γ should be determined from the fit of data.

Having in hand the moments (12,9,5, 8) and following the method [4, 5], we can write the structure function xF_3 in the form:

$$xF_3^{N_{max}}(x, Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{j+2}^{NS}\left(Q^2\right), \tag{12}$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha, \beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers of x:

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) x^j. \tag{13}$$

The quantities N_{max}, α and β have to be chosen so as to achieve the most fast convergence of the series in the r.h.s. of Eq.(12) and to reconstruct xF_3 with the accuracy required. Following the results of [5] we use $\alpha = 0.12$, $\beta = 2.0$ and $N_{max} = 12$. These numbers guarantee accuracy better than 10^{-3} .

Eq. (12) could be applied for reconstructing SF $F_2(x, Q^2)$ for $0.3 \leq x$ and with eq. (1,4) for TMC taken into account.

The parameters A, b, c, γ and parameter Λ are determined by fitting experimental data. We also consider $M^{eff.}$ as a free parameter. It should be noted that the parameters a, b, c and γ depend on Q_0^2. We have used experimental points with $Q^2 > 5GeV^2$ for fitting, in order to avoid high–twist effects and chose $Q_0^2 = 10GeV^2$.

The results of concrete calculations made for SF measured in experiments on different targets are presented in Table I.

For the hydrogen target $M^{eff.}$ reproduces the value of the proton mass. For the iron target the effective mass $M^{eff.}$ is twice the nucleon mass. The data of the SKAT collaboration on a target which consists of a mixture of Neon and Hydrogen are not precise enough to determine the value of Λ. So following [7] we have fixed $\Lambda = 200MeV$ and found the value of $M^{eff.}$ a little bit higher than for the hydrogen target. The increasing effective mass of a nuclon on the nucleus target takes place for a nonsinglet fit both for F_2 and xF_3 SF. It also takes place both for the leading and next to leading order QCD (see result for xF_3 data of CCFR Table 1.). The large value of $M^{eff.}$ found in the QCD fit of data of DIS on nucleon target could be considered as indirect evidence of the existence of multiquark clusters [10, 11, 12, 13] or a few–nucleon correlation in a nucleus [14]. It is also compatible

with the measured SF at $x > 1$ on DIS of leptons on the nucleus target [15].

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Collaboration Reaction	Ref.		Λ $[MeV]$	$\chi^2_{d.f.}$	$M^{eff.}$ $[GeV]$
BCDMS μp F_2	[6]	$0.35 < x$	130 ± 4	183/223	0.88 ± 0.14
SKAT $\nu Ne, p$ xF_3	[7]	$0.05 \leq x$	200 (fix.)	25.3/30	1.42 ± 0.71
EMC μFe F_2	[8]	$0.30 < x$	106 ± 26	45.3/45	2.08 ± 0.16
CCFR νFe F_2	[9]	$0.275 \leq x$	146 ± 12	37.9/81	1.76 ± 0.09
CCFR νFe xF_3	[9]	$0.015 \leq x$	64.7 ± 21	81.8/81	2.04 ± 0.18
CCFR νFe xF_3 NLO	[9]	$0.015 \leq x$	116 ± 30	73.4/81	1.83 ± 0.20

Table I. The summary of various determinations of the $M_n^{eff.}$.